is true, but the cause may lie either in the fact that T_0 is constant only for similar fluids—that is for fluids of the same chemical type, or in the difficulty in identifying the density of Laplace's theory with a particular physical property.

Comparing equations (3) of the paper preceding and (12) we get

$$\rho_0{}^2 T_0 = \frac{(\tau-d)}{v^{2/3}}\kappa.$$

If the density be taken as a molecular quantity then $\rho = 1/v$ and $T_0 = (\tau-d)\,\kappa V^{-\frac{1}{3}}$. From this we can derive equation (4) of the preceding paper as the expression for the interfacial tension.

The expression $T_{AB} = (\rho_A - \rho_B)^2\, T_0$ is now seen to be wrongly derived, the false assumptions being the identity of κ_A with κ_B, and of $(\tau_A - d)$ with $(\tau_B - d)$. And for a similar reason the expression $T'_{AB} = \rho_A \rho_B T_0$ is inadmissible.

A Simple Method of Finding the Approximate Period of Stable Systems.

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In practical engineering work it is often a great convenience to be able to find the period of a structure, the calculation of which, by ordinary mathematical processes, would be difficult or even impossible. To find the period of a structure for any particular mode of vibration involves a knowledge of its stiffness (regarded as a spring) and of the distribution of the mass, but if the latter is known, even approximately, a knowledge of the period gives the stiffness, and the deflections for a given load can be found by simple arithmetic.

In nearly every case likely to occur in practice a stable structure can be represented, as far as its elastic displacements are concerned, by an equivalent pendulum, a pendulum, that is, which has the same period as the particular mode of vibration under consideration, and an effective mass equal to that part of the mass of the structure which is subject to vibration, but concentrated at what, for the present purpose, may be called the centre of oscillation.

The proposition on which the simple determination of period above referred to depends is as follows:—

Let the length of the equivalent pendulum be l, and let σ be the stiffness of the structure, so that the force required to cause a displacement x measured at the centre of oscillation is σx. Now let x_1 be taken so that $\sigma x_1 = \mathrm{W}$ where W is the weight of the structure. The period τ is given by either of the equations $\tau = 2\pi\sqrt{(l/g)}$ or $\tau = 2\pi\sqrt{(\mathrm{W}/g\sigma)}$,

$$\therefore\ l/g = \mathrm{M}/\sigma, \quad \text{and since} \quad \sigma = \mathrm{M}g/x_1, \quad \mathrm{M}/\sigma = x_1/g, \quad \therefore\ x_1 = l.$$

That is the deflection x_1 (the deflection, namely, caused by a force equal to the weight of the structure) is equal to the length of the pendulum which has the period of the structure.

This statement applies accurately to all cases where the mass is concentrated at one place, or where the structure moves without deformation or rotation, and in which, consequently, the mass may be supposed to exist only at the centre of inertia. In other cases the application is approximate, but by a judicious choice of position at which the deflection is measured, or of the assumed distribution of the mass, the approximation may be made very close.

A few examples will indicate the uses to which the proposition may be turned—

(1) A rod or pole, ballasted to float upright in a fluid, has the same up and down period as a pendulum whose length is the depth of statical immersion. The only correction to be applied here is that for the inertia of the fluid set in motion; this is known for many forms and tends towards zero when the rod is long and thin.

(2) A spiral spring which is compressed through the unit of length by the action of N units of weight is loaded with a weight W. The compression due to the load is $\mathrm{W}/\mathrm{N} \times$ unit of length, and the approximate period is $2\pi\sqrt{(\mathrm{W}/\mathrm{N}g)}$. The correct period can be found by adding one-third of the weight of the spring to the load.

(3) A girder of known weight is deflected N units of length by a central or terminal load equal to its own weight. The approximate period when unloaded is $2\pi\sqrt{(\mathrm{N}/g)}$. If the section of the girder is uniform, a closer approximation may be obtained if the deflection is measured at a quarter of the half length from the centre in the case of a girder supported at both ends, or a quarter of the length from the free end when fixed at one end only.

(4) A ship which has a mean draught N feet of water has pitching period dependent on the draught and on its moment of inertia about the axis of pitching. If the ship had no rigidity and behaved as if made up of a number of detached vertical cross-sections floating at the mean draught, the pitching period would be nearly $2\pi\sqrt{(\mathrm{N}/g)}$. A correction is required for rigidity and also for the inertia of the water involved in the motion. The

first of these corrections is small for ships of ordinary proportions, and decreases with the ratio of draught to length. The second increases the period (as calculated from the formula) by something like 10 per cent. The precise amount depends on the ship's "lines."

(5) If a vertical column when struck is found to give a note of a certain pitch, the principle here used allows of the immediate determination of its flexure under any given lateral force, and if the nature and dimensions of the column are known, the pitch under no load can be determined from first principles. As the load on the ends increases, the natural period increases also, becoming infinite when the unstable condition is approached and hence the observation of the actual pitch gives a measure of the load which is borne.

I first noticed the relation here stated in 1878, and since that time have found it of great use in almost countless investigations, but except for one case (involving the same principle) referred to by D. Bernoulli it does not, so far as I am aware, appear to have been stated in any publication.

[*April* 10, 1913.—Dr. Schuster has pointed out to me that there were mistakes in the original text of examples (2), (3), and (4). These have now been corrected.—A. M.]